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## REPORT DOCUMENTATION PAGE (SF298) (Continuation Sheet)

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## FINAL SHAPE OF PRECISION MOLDED OPTICS: PART II—VALIDATION AND SENSITIVITY TO MATERIAL PROPERTIES AND PROCESS PARAMETERS

Balajee Ananthasayanam<sup>1</sup>, Paul F. Joseph<sup>1</sup>, Dhananjay Joshi<sup>1</sup>,  
Scott Gaylord<sup>2</sup>, Laetitia Petit<sup>2</sup>, Vincent Y. Blouin<sup>2</sup>,  
Kathleen C. Richardson<sup>2</sup>, Daniel L. Cler<sup>3,4</sup>, Matthew Stairiker<sup>5</sup>,  
and Matthew Tardiff<sup>5</sup>

<sup>1</sup>Department of Mechanical Engineering, Clemson University,  
Clemson, South Carolina, USA

<sup>2</sup>School of Materials Science and Engineering, COMSET Clemson University,  
Clemson, South Carolina, USA

<sup>3</sup>Benét Laboratories, Watervliet, New York, USA

<sup>4</sup>U.S. Army RDECOM/ARDEC, Picatinny Arsenal, New Jersey, USA

<sup>5</sup>Edmund Optics, Pennsburg, Pennsylvania, USA

*In Part I of this study a coupled thermo-mechanical finite element model for the simulation of the entire precision glass lens molding process was presented. That study addressed the material definitions for the molding glass, L-BAL35, computational convergence, and how the final deviation of the lens shape from the mold shape is achieved for both a bi-convex lens and a steep meniscus lens. In the current study, after validating the computational approach for both lens types, an extensive sensitivity analysis is performed to quantify the importance of several material and process parameters that affect deviation for both lens shapes. Such a computational mechanics approach has the potential to replace the current trial-and-error, iterative process of mold profile design to produce glass optics of required geometry, provided all the input parameters are known to sufficient accuracy. Some of the critical contributors to deviation include structural relaxation of the glass, thermal expansion of the molds, TRS and viscoelastic behavior of the glass and friction between glass and mold. The results indicate, for example, the degree of accuracy to which key material properties should be determined to support such modeling. In addition to providing extensive sensitivity results, this computational model also helps lens molders/machine designers to understand the evolution of lens profile deviation for different lens shapes during the course of the process.*

**Keywords:** Aspherical glass lens; Coupled thermo-mechanical numerical simulation; Micron deviation; Structural relaxation; Temperature dependent material parameters; Viscoelasticity

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Address correspondence to Paul F. Joseph, Department of Mechanical Engineering, Clemson University, Clemson, SC 29634-0921, USA. E-mail: jpaul@clemson.edu

## INTRODUCTION

Part I of this two-part series included a discussion of the process of precision molding of glass optics, a review of the pertinent literature, the associated finite element model (FEM) details, and the glass and mold material definitions. From the point of view of the current study, the most important issue is the profile deviation, as illustrated in Figure 2 of Part I. Optical designers require the actual lens shape to be less than one micron from the target shape, while in practice typical deviations from the mold shape can easily be 20 microns or more. Therefore, it is desirable to have an efficient method to determine the mold shapes to achieve a desired lens shape to within sufficient accuracy. The current trial and error approach of mold compensation is both costly and time consuming. This has motivated the development of computational approaches to create a compensated mold shape taking into account process parameters and the complex thermo-mechanical behavior of glass. A major obstacle to modeling is the lack of availability of material properties of glass that are relevant to the molding process. The objective of this study is to use computational analysis to identify the key material properties and process parameters that have an effect on deviation.

### Literature

The research related to sensitivity of lens profile deviation to the variation in material properties and process parameters in precision lens molding is presented in this section. For a more general review of the precision lens molding process, refer to Part I of this two-part series. The first published simulation study on lens profile deviation is by Yi and Jain [1], who compared a computational prediction of deviation with experimental data. In that article glass was modeled as purely viscous and thermal expansion was accounted for with a constant coefficient of thermal expansion (CTE). The experimental results indicated that this level of modeling was not sufficient, so in a later study, Jain and Yi [2] improved their thermal expansion assumption by making use of the TNM model to account for the so-called structural relaxation behavior of glass [3]. The TNM model parameters were taken from Scherer [3] and correspond to those of window glass. The mechanical behavior of the glass used by Jain and Yi [2] was modeled as a viscous material in the pressing stage and as a viscoelastic material (single Maxwell element) in the later cooling stages. The predicted curve geometries using the TNM model and a constant CTE produced entirely different results. Hence they concluded that structural relaxation behavior plays an important role on the final geometry. A sensitivity analysis was also performed to study the influence of cooling rate on final lens geometry and it was concluded that the cooling rate does not have a significant effect on the lens geometry.

The goal of this article is to quantify the effect on deviation of several material properties and process parameters by using the computational model presented in Part I of this two-part series. Such a computational mechanics approach has the potential to replace the current trial-and-error, iterative process of mold profile design to produce glass optics of required geometry. This sensitivity analyses will not only identify the parameters that have little or no effect on the final size/shape, but also identify some key material properties and substantiate the need to obtain them more accurately through experimentation.

## RESULTS

In this article the axi-symmetric finite element model (FEM) described in Part I was used in two validation studies and extensive sensitivity results. These studies involved the following three lens geometries:

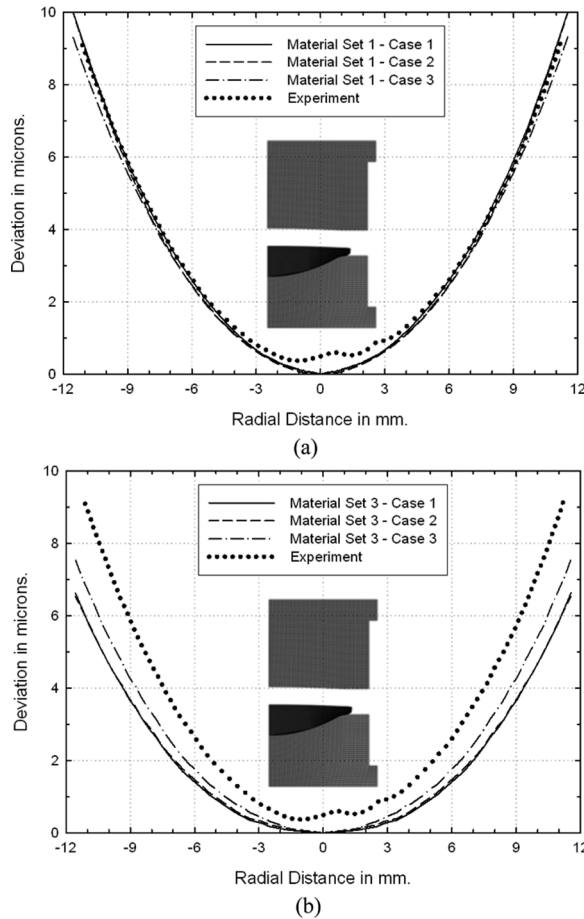
- Bi-convex lens from Part I used for the first validation study (Figure 1) and several sensitivity studies (Figures 4–15, 17). The process parameters in Figure 8 of Part I apply. Only the deviation on the lower aspherical surface is considered. This lens is also referred to as “the validation” case in some of the figures.
- Steep meniscus lens from Part I that was introduced to reveal complications with stress relaxation behavior was used in several sensitivity studies (Figures 9, 10, 12, 16, 17). The process parameters in Figure 8 of Part I apply. Only the deviation of the lower aspherical surface is considered.
- Steep meniscus lens, very similar to the shape of the above lens, was used only in a second validation study (Figure 3). The process parameters are different from those in Figure 8 of Part I and this case was heated and pressed in a vacuum. Deviations on the lower aspherical surface and the upper spherical surface are considered.

After the validation studies are presented, sensitivity results are presented for both the bi-convex lens and the steep meniscus lens. While the validation studies involve both Material Sets 1 and 3, which were introduced in Table 3 of Part I, the sensitivity analyses are based on Material Set 1.

### Validation of Model with Experiments

Following Part I of this study, the deviation is defined as the difference between the final lens shape from that of the mold at room temperature, i.e., the “desired profile” in Figure 2 of Part I is the mold shape. All results correspond to materials defined by the properties in Tables 1–3 of Part I, which were obtained from experimental data and from the literature. Although the molding glass is intended to be L-BAL35, if one or more of the material characterizations of the various input parameters are not accurate, then it is possible to generate results that do not agree with the actual molding of this glass type. In this regard, stress relaxation of the glass and the gap conductance heat transfer parameters are of the most concern. In the validation results that follow, due to this uncertainty, both the stress relaxation behavior and the gap conductance parameters are used in a sensitivity analysis.

The deviations from the experiment and from the simulations for the bi-convex lens, introduced in Figure 11 of Part I, are presented in Figure 1. The results include stress relaxation defined by Material Set 1 in Figure 1a and by Material Set 3 in Figure 1b. Although Material Set 3 is believed to be the more representative of L-BAL35, the temperature/viscosity relationship is not known with certainty. Concerning gap conductance, there are two issues making it difficult to assign values. First, is the rough profile of the experimental deviation profile in Figure 1, which is addressed in the next paragraph. The second point is that in the experiment the lens was soaked in a nitrogen environment, but pressed in a vacuum. Because of the vacuum, the gap conductance parameters used previously in Part I



**Figure 1** Comparison of experimental and simulated deviation for the bi-convex lens introduced in Figure 11 of Part I. Figure 1a – Material Set 1 Cases: 1) Lens soaked in N<sub>2</sub>, 2) Lens Soaked in N<sub>2</sub> and pressed in a Vacuum with gap radiation, 3) Lens soaked in N<sub>2</sub> and pressed in a Vacuum without gap radiation. Figure 1b – Material Set 3 Cases: 1) Lens soaked in N<sub>2</sub> and pressed in a Vacuum with a gap radiation function 1, 2) Lens Soaked in N<sub>2</sub> and pressed in a Vacuum with gap radiation function 2, 3) Lens Soaked in N<sub>2</sub> and pressed in Vacuum without gap radiation. See Table 1 for heat transfer details.

must be changed to reflect an increased resistance to heat transfer, since there is no longer a gas through which heat can be conducted. Lacking the experimental capability to measure gap conductance, the adjusted parameters were determined based on the press time. In general the press time is affected by molding force, molding temperature and mold/glass friction. Therefore, in using temperature alone to adjust the press time, it was assumed that the force reading from the machine is correct as well as the measurement of the friction coefficient. The adjustment of the gap conductance parameters also requires that radiation effects are changed. The uncertainty of exactly how to modify these parameters gives rise to the different combinations presented in Figure 1, which are defined in Table 1.

**Table 1** Heat transfer parameters used in the sensitivity analysis associated with the validation of the bi-convex lens presented in Figure 1

Heat transfer parameters for validation of Bi-convex lens					
Cases in Figure 1	Soaking stage ( $N_2$ )		Pressing stage (Vacuum)		Gap Radiation Function*
	$k$ (W/m·K)	$h_{\max}$ (W/m <sup>2</sup> ·K)	$k$ (W/m·K)	$h_{\max}$ (W/m <sup>2</sup> ·K)	
Material Set 1 (Figure 1a)					
Case 1	0.04	5000	0.04	5000	#2
Case 2	0.04	110	0.001	60	#1
Case 3		5000		2500	–
Material Set 3 (Figure 1b)					
Case 1	0.04	40	0.001	20	#1
Case 2		80		30	#2
Case 3		110		50	–

\*Gap radiation functions defined below.

Gap radiation function	Emissivity of the mold	Emissivity of glass	View factor	Clearance (m)
#1	0.46	0.46	1	0
			1	6e-3
			0	10e-3
#2	0.15	0.85	1	0
			1	2e-3
				3e-3

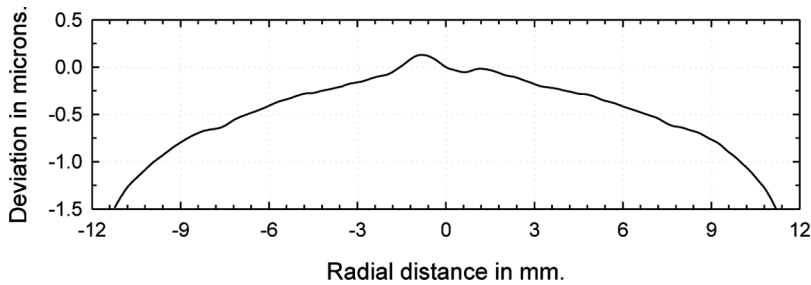
The unusual profile near the center of the measured lens is due to an imperfectly ground mold, which makes it difficult to define a zero point. Since the Case 1 result in Figure 1a is so close to the data, the experimental curve is matched at an arbitrarily selected point, which was taken to be 6.93 mm to the right of center. This positioning of the data was also used in Figure 1b. In order to better explain the unusual profile near the center of the measured lens, the difference between the measured mold shape and the mathematically defined mold shape,

$$Z(Y) = \frac{CY^2}{1 + \sqrt{1 - (1+k)C^2Y^2}} + EY^4 + FY^6 + GY^8 + HY^{10}, \quad (1)$$

is presented in Figure 2. The constants defining the mathematical shape in (1) are listed in Table 2. Such a profile helps to justify the large uncertainty in the gap conduction parameters presented in Table 1 and why such low values of  $h_{\max}$  are considered in the sensitivity analysis, especially during the soak stage for Material Set 3.

Now that the background information has been provided to justify and explain the various cases in Figure 1, the results will be explained. The three simulation curves in Figure 1a correspond to: case 1) very little gap radiation during press, case 2) full gap radiation during soak and press and case 3) the limiting case of no gap radiation during soak and press. It is observed that results for all three





**Figure 2** Experimental mold deviation measured with respect to the target aspherical surface defined by  $Z(Y)$  in Equation (1) with parameter values given in Table 2.

of these cases match the data very well; however, the viscosity in Material Set 1 is believed to be too high for L-BAL35. The unusual profile near the center of the lens is due to an imperfect mold surface, which is addressed in the paragraph below. In Figure 1b the results correspond to the more correct viscosity behavior from Material Set 3. Once again three different heat transfer options are considered, all leading to the correct press time. In this set of results the deviation is lower than the Material Set 1 results in Figure 1a. This occurs since the lower viscosity of Material Set 3 requires less heating reaching the target press time. A lower peak temperature results in less deviation. Although the results are good, it appears as though in addition to gap conductance and stress relaxation, there could be other factors that are not characterized precisely. It is very likely that the correct behavior for both viscosity and gap conductance is somewhere between the parameters used in Figures 1a and 1b. The important point is that the trends are correct and the results are close.

The next validation study is for a steep meniscus lens, slightly different in shape from that of Figure 13 in Part I. The material properties used in this simulation are given in Tables 1–2 and Material Set 3 of Table 3 of Part I. The process parameters, however, are different from those presented in Figure 8 of Part I, and most notably, both the soak and press stages were done in a vacuum. Because of heating and pressing in a vacuum, the gap conductance parameters used previously must again be changed to reflect an increased resistance to heat transfer. The results of the second validation study are presented in Figure 3 and include deviations of both the upper and lower lens surfaces. In this case it was necessary

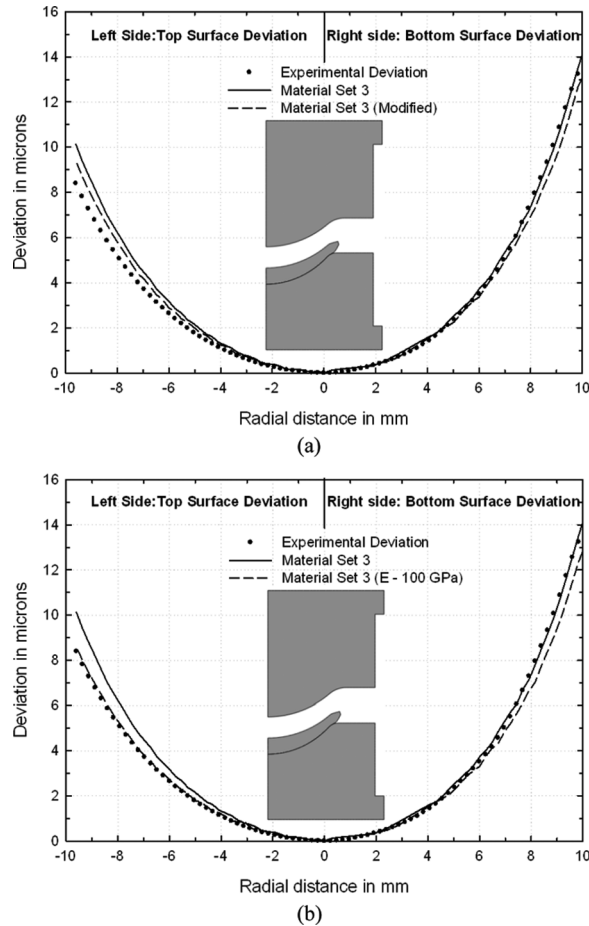
**Table 2** Aspherical surface parameters defined in Eqn. (1)

Aspherical surface parameters	
$C = 1/R$	0.045140613
$k$	−2.271309
$E$	1.954456e-5
$F$	−1.756349e-8
$G$	2.597437e-11
$H$	−2.414065e-14

to use Material Set 3 to represent L-BAL35 due to the more realistic viscosity, TRS behavior and elastic modulus.

The hypothetical molding glass represented by Material Set 1 produces a deviation that over predicts the deviations by almost a factor of two for both surfaces.

Similar to Figure 1, there are two parts, i.e., Figures 3a and 3b, to explore two features of stress relaxation from a sensitivity point of view. In Figure 3a, the deviation profiles using a multiple term prony series to represent the shear and hydrostatic relaxation functions from Material Set 2, are included as a modified Material Set 3 case. It appears that a single-term prony series in shear and bulk

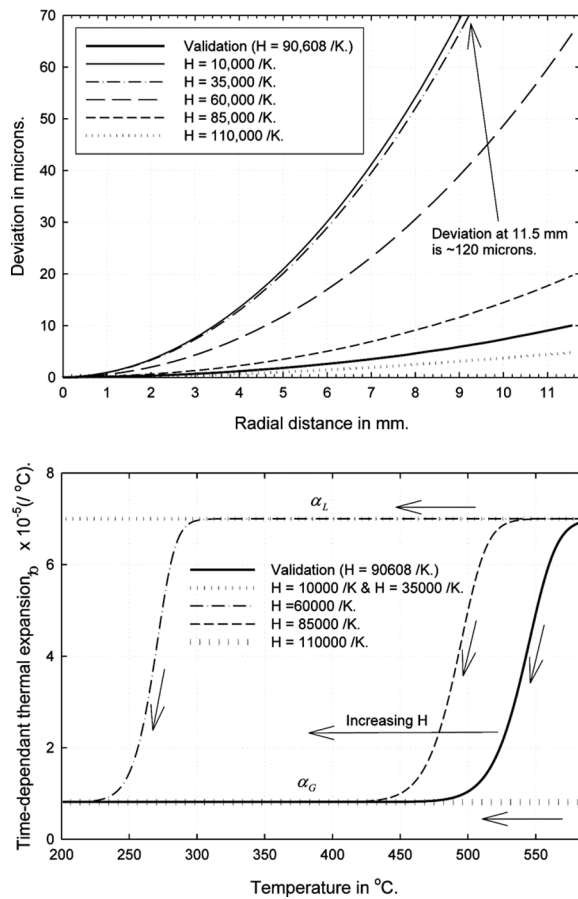


**Figure 3** Deviation from simulation compared with the experimental deviation for the steep meniscus lens shape shown as an insert. The top surface deviation (spherical) is plotted on the left side, while the bottom surface deviation (aspherical) is plotted on the right side. Material Set 3 is used with two modifications. In Figure 3a, the modified set includes the shear and hydrostatic relaxation functions of Material Set 2, which allows comparison between a single and a multiple term prony series. In Figure 3b the modified set makes use of a Young's modulus of 100 GPa above 560 degrees instead of 10 GPa.

modulus is sufficient in this case. It is also known that an elastic representation of the bulk modulus produces the same results. This is a significant result since the time dependence of the stress relaxation behavior is difficult to obtain. In Figure 3b the deviation profiles are included for a stiffer glass at high temperature by modifying Material Set 3 to have a Young's modulus of 100 GPa above 560°C instead of 10 GPa. It has already been shown in Figure 14 of Part I that a lower value of  $E = 0.8$  GPa will make a more significant difference.

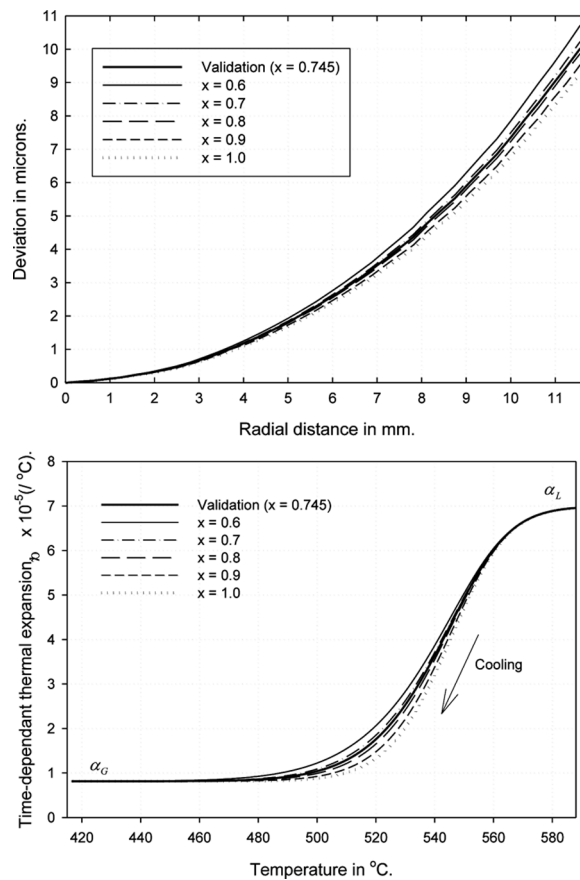
### Effect of Structural Relaxation of Glass and Thermal Expansion of the Molds

Structural relaxation, which in this computational study is understood to be temperature history dependent thermal expansion, is the most basic reason for



**Figure 4** Sensitivity of the activation energy constant on deviation is shown in the upper plot. In the lower plot the instantaneous thermal expansion coefficient is presented as a function of temperature for various activation energy constants for a uniformly cooled volume of glass for a cooling rate of approximately 25°C/min. The arrows in the lower figure indicate the cooling direction.

deviation in the lens molding process. Since glass typically has a higher average CTE than the mold material, the deviation will usually be positive on the convex and concave sides of a lens, i.e., a sphere of glass when cooled will have a smaller radius. The four parameters that define the structural relaxation model will now be used in a sensitivity analysis. This includes the activation energy constant,  $\Delta H/R$ , the nonlinearity parameter,  $x$ , the time constant parameter,  $\tau_0$ , and the Kohlrausch shape function,  $\beta$ . The sensitivity analyses for these parameters are presented in Figures 4–7, respectively. The deviation due to the effect of structural relaxation is most easily understood by considering the lower portion of these figures, where for the purpose of illustration, the instantaneous thermal expansion coefficient is presented as a function of temperature for a uniformly cooled volume of glass. Given that volume change is proportional to the integral of thermal expansion with respect to temperature, the more area under the curve, the more volume change due to thermal expansion. Although the curves in the lower portion of Figures 4–7 apply to a uniformly cooled volume of glass, within the computational model used for the



**Figure 5** Same as Figure 4 for the non-linearity parameter,  $x$ .

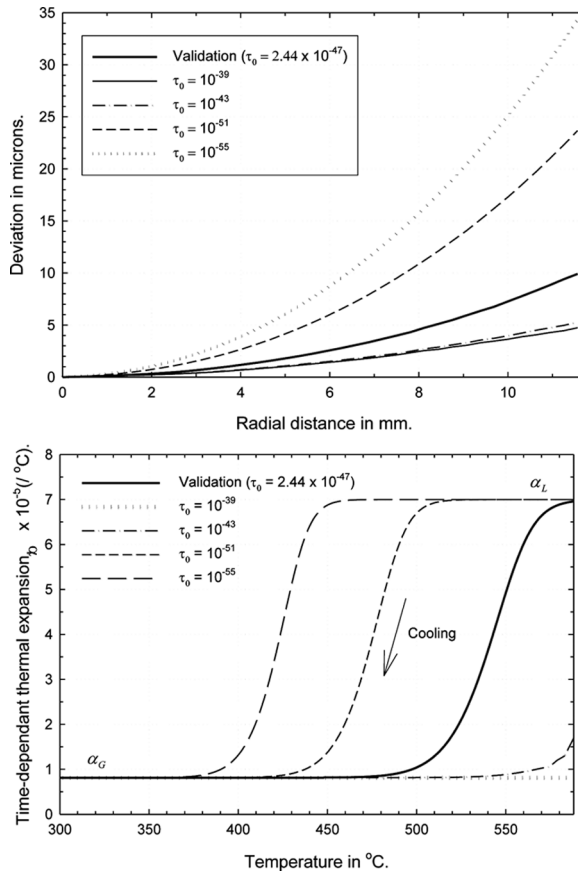


Figure 6 Same as Figure 4 for the time constant,  $\tau_0$ .

upper plots in these figures, each material point experiences a different temperature as a function of time and therefore has a different cooling rate.

This set of four figures shows the very strong effect on deviation of the activation energy parameter,  $\Delta H/R$  in Figure 4 and the time constant parameter,  $\tau_0$ , in Figure 6. Clearly glasses with different values of these parameters must be compensated differently. From a computational point of view, it is essential to have these structural relaxation parameters accurately defined.

The thermal expansion coefficient of the mold material, which is modeled as a linear elastic material and hence does not display structural relaxation, is also an important parameter. In Figure 8 the effect of the thermal expansion coefficient clearly shows a significant effect on deviation. In fact, if the thermal expansion of the mold is large enough, the deviation can become negative.

### Effect of Viscoelastic Behavior of Glass and Residual Stresses

In Part I the effect of stress relaxation was shown to be more important for a steep meniscus lens than for a bi-convex lens. Therefore, in this section both

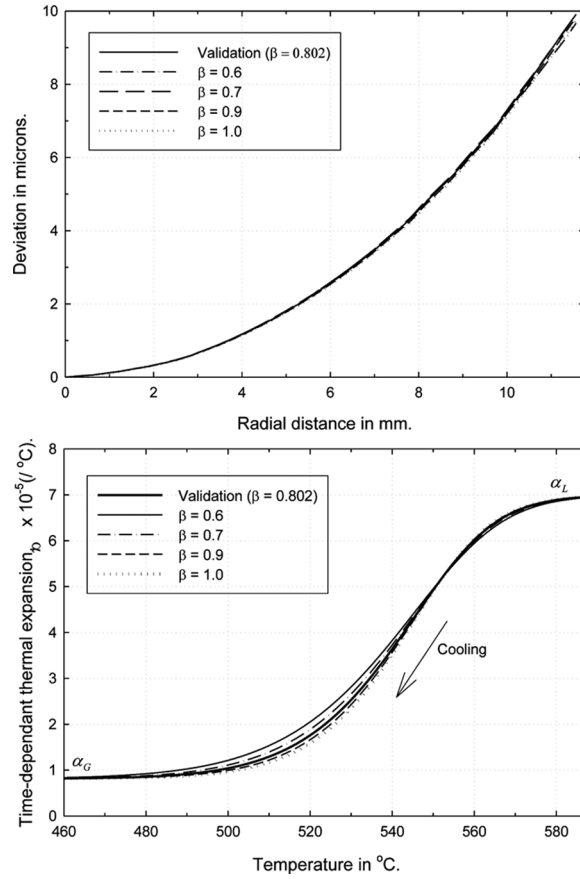
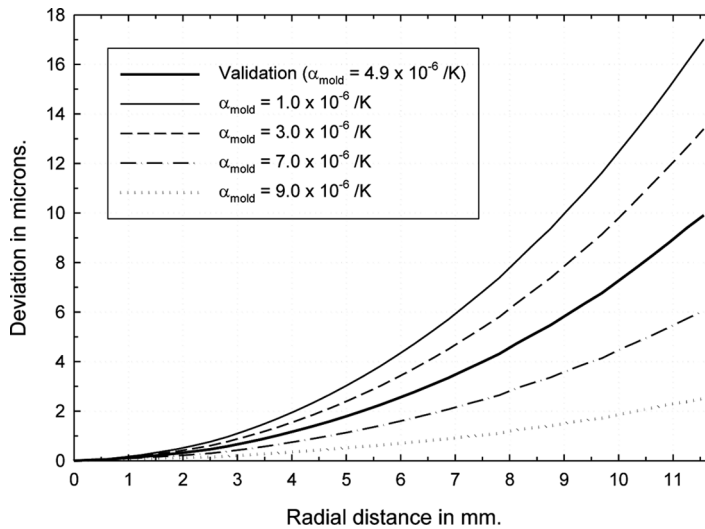


Figure 7 Same as Figure 4 for the Kohlrausch factor,  $\beta$ .

lens shapes will be considered in the analysis. The two primary features of stress relaxation to consider are the shape of the master curves and the temperature dependence as defined by the TRS shift. The study starts with the sensitivity of deviation to the shape of the master stress relaxation curve for shear. Care was taken to ensure that the equilibrium viscosity at the reference temperature remained constant during this study. The shear relaxation function,  $G_1(t)$  is related to the equilibrium viscosity of the glass,  $\eta$ , by the relation,

$$\tau_{avg} = \sum_{i=1}^n w_i \tau_i = \frac{\eta}{G_0} \quad (2)$$

where ' $\tau_{avg}$ ' is the weighted average of the relaxation times. When the prony series was altered under the constraint given in (2), the deviation did not change significantly for either lens shape. The deviation was also insensitive to reasonable changes in the hydrostatic relaxation function,  $G_2$ . The above conclusions are for the process conditions presented in Figure 8 of Part I. In particular, the slow cooling period has a duration of 320 seconds. Later it will be shown that as this period

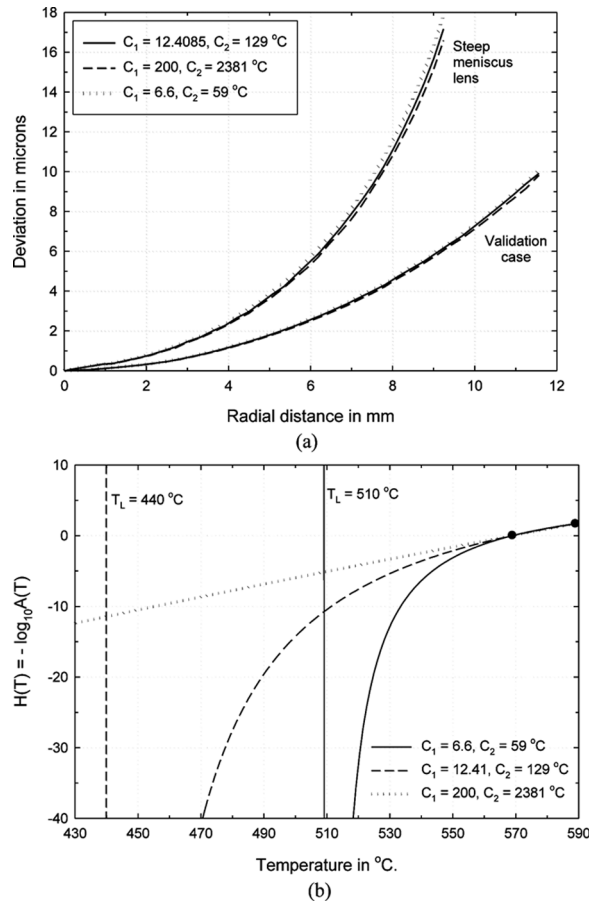


**Figure 8** Sensitivity of the thermal expansion coefficient of the mold on the deviation of the bi-convex lens.

becomes shorter, the deviation becomes more sensitive to the stress state in the lens when the gap opens, and therefore possibly to the shape of the master curve.

The stress state is dependent on the TRS behavior, since this defines the way stresses relax. As a preliminary study to see how the TRS behavior affects deviation, the reference temperature,  $T_R$ , of the glass is shifted by up to 20 degrees, while keeping everything else the same. As such, this numerical experiment corresponds to the case where temperature measurements for all specimens used to characterize stress relaxation are off by the same temperature. When a lens is pressed with such a glass, and everything except for press time is kept the same as for the glass with its original reference temperature, there is essentially no change in deviation if the glass is heated to the same molding temperature. The press time must be changed to achieve the same center thickness, i.e., if the reference temperature is lowered, the glass is easier to deform so the press time must decrease. These results are important since it is non-trivial to know precisely the temperature of a glass specimen used to characterize stress relaxation. However, in Figure 1 it was shown that when the shift is not uniform, for example comparing the TRS behaviors of Material Sets 1 and 3, non-negligible changes in deviation occur. Therefore, additional cases of a non-uniform shift are considered in the next sensitivity study.

In Figures 9 and 10 two additional TRS studies have been conducted and each study involves three TRS behaviors. In Figure 9(b), all the three TRS behaviors behave the same way in the temperature range 569°C–589°C, whereas in Figure 10(b) all the three TRS behaviors are equal only at 589°C. In Figure 9, where the TRS behaviors are the same for the temperature range, 569°C–589°C, there is very little change in deviation. However, in Figure 10, where the TRS behaviors only match at 589°C, there is a significant change for both the steep meniscus lens *and* the bi-convex lens. Therefore, the deviation is most sensitive to the TRS behavior



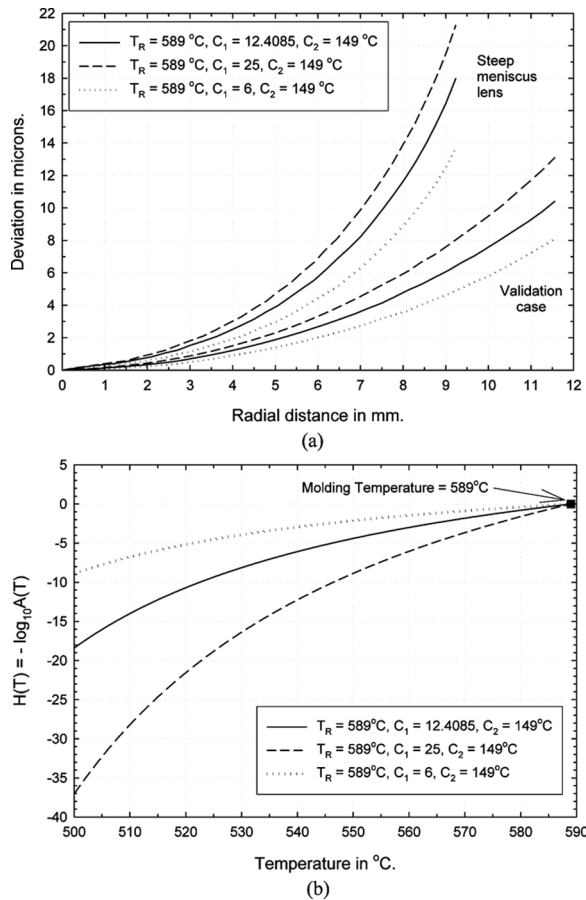
**Figure 9** (a) Sensitivity of TRS behavior on deviation for the cases shown in the lower plot where  $TR = 569^\circ\text{C}$ . (b) Illustration of different TRS behaviors while requiring the shift factor curve,  $A(T)$ , to pass through the two points indicated.

at temperatures near the molding temperature ( $569^\circ\text{C}$ – $589^\circ\text{C}$ ), which is where stress relaxation effects are most dominant.

Referring to Figure 10, it is seen that if relaxation times increase at a slower rate (dotted curve), the viscosity of the glass also increases at a slower rate and hence the material flows more easily. Since the material is able to flow, it can better accommodate the change in shape driven by cooling, thereby decreasing the deviation. Conversely, if relaxation times increase at a faster rate (dashed curve) the deviation increases since the glass viscosity increases at lower temperature, which reduces the time for the glass to adjust. Referring to Figure 9, the three different TRS behaviors differ only below  $569^\circ\text{C}$  and only small changes in deviation occur for both lens types. This implies that the TRS behavior at temperatures near the molding temperature is very crucial to predicting the deviation accurately.

The important part of the study presented in Figures 9 and 10 is that the press time is constant, because the viscosity at the molding temperature of  $589^\circ\text{C}$  is





**Figure 10** (a) Sensitivity of TRS behavior on deviation for the cases shown in the lower plot for both lens shapes (see also Table 3). (b) Temperature dependence of the different shift factors,  $A(T)$ , while maintaining  $TL = 440^\circ\text{C}$ .

constant. Therefore, the temperature distributions within the lenses are essentially the same for the entire process, and any difference in deviation for each lens type must be attributed to stress relaxation behavior. As the lens experiences temperatures above  $569^\circ\text{C}$  only before the gap opens, it is interesting to study the contributions to the total deviation before, during and after the gap creation stage. This breakdown of the Figure 10 results is presented in Table 3. The profiles of the deviations for  $C_1 = 12.41$  are presented in Figure 16 of Part I. The results in Table 3 show the effect of stress relaxation is present for both lens types, during the slow cooling stage when the lenses are supported as shown in Figure 17 of Part I. When the pressing force of 2000 N is reduced to the 500 N maintenance force, which marks the beginning of the slow cooling stage, the lens has the opportunity to change shape and develop a deviation. This non-negligible deviation, which evolves slowly during this 320 s slow cooling period, is driven by the stress state. These results therefore

**Table 3** Comparison of changes in deviation before, during and after gap creation for the TRS behaviors indicated in Figure 10.

TRS Behavior $T_R = 589^\circ\text{C}$ $C_2 = 149^\circ\text{C}$	Deviation before, during and after gap creation ( $\mu\text{m}$ )							
	Steep meniscus lens				Bi-convex lens			
	Before	During	After	Total	Before	During	After	Total
$C_1$								
6	2.6	4.6	6.5	13.7	3.7	-0.3	4.6	8.1
12.41	6.5	4.9	6.5	18.0	5.9	-0.1	4.6	10.4
25	8.3	6.5	6.5	21.3	7.6	1.1	4.3	13.1

The 0.15 mm gap is created during a 19 second period that separates the slow cooling stage, which comes before the gap is created, from the fast cooling stage, which comes after.

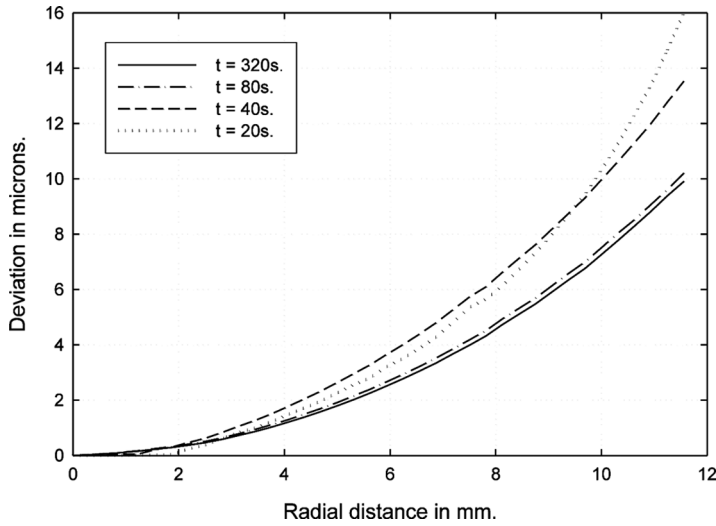
show the importance of an accurate characterization of the TRS behavior of the glass.

From the results presented in Figure 10, clearly deviation is sensitive to the temperature dependence of the mechanical behavior of glass. In Part I of this study it was shown that the stress state in the bi-convex lens did not appear to be a major factor for shape change due to the manner in which the lens was supported when the gap opened. Because temperature is so important, one final set of results in this section will study the effect on deviation of the time at which the gap opens, while maintaining the same slow cooling rate. Referring to Figure 8 in Part I, this corresponds to opening the gap at different temperatures. For example, for the  $t = 320\text{ s}$  case the gap was created when the temperature at the end of the slow cooling stage was around  $450^\circ\text{C}$ , whereas for  $t = 20\text{ s}$ , the gap would be created at approximately  $579^\circ\text{C}$ .

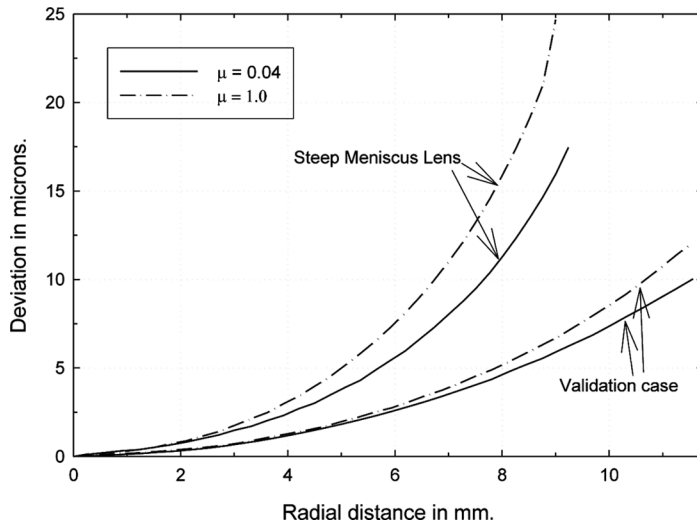
As shown in Figure 11 the deviation was sensitive to a short maintenance force period. This figure not only shows the importance of applying the maintenance force, but also identifies a time beyond which no further change in deviation occurs. For the case of the bi-convex lens, it appears as though the 320 second slow cooling period can be reduced to 80 seconds in order to reduce the processing time. This conclusion is strongly dependent on the accuracy of both the master curve and the shift factor that characterize stress relaxation. This conclusion is also dependent on the molding temperature. Since the stress decays more slowly at lower temperature, this minimum time of 80 seconds would need to increase if the molding temperature decreases.

### Effect of Friction at the Interface

The shear stresses applied to the surface of the lens during the pressing phase are a function of both friction and the shapes of the preform and the molds. Given that the lens geometries are fixed in this study, this effect can be artificially created by using different friction scenarios. Furthermore, one way to determine the effect of residual stresses on deviation is through numerical experiments involving friction, because by changing the coefficient of friction, the residual stress state is changed. In Figure 12 two different friction coefficients were used for the bi-convex and the steep meniscus cases. The baseline friction coefficient of  $\mu = 0.04$ , corresponds to



**Figure 11** Comparison of final deviation for various durations of the slow cooling stage, while keeping the maintenance force equal to 500N. The case ' $t = 320$  s' corresponds to the validation case presented in Figure 1. The cooling rate in all these simulations was kept constant at  $25.875^{\circ}\text{C}/\text{min}$ .



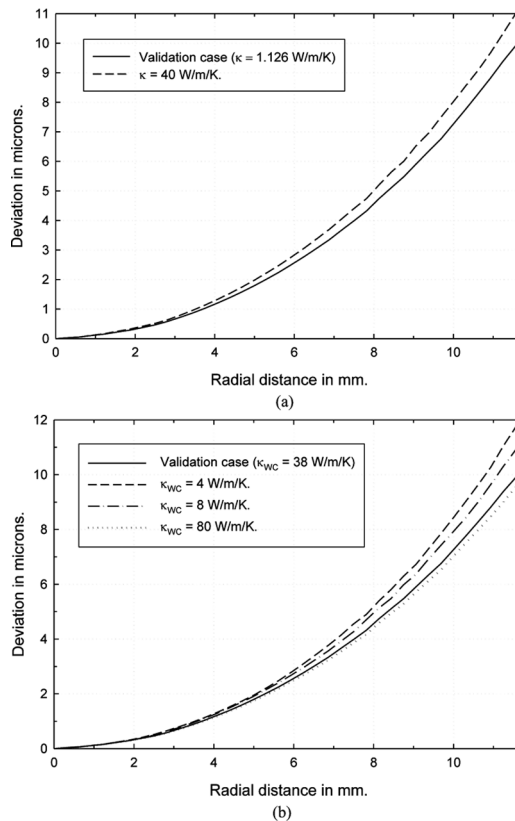
**Figure 12** Sensitivity of the coefficient of friction at the glass/mold interface on final deviation for both lens shapes.

very low friction, while  $\mu = 1.0$  corresponds to very high friction. Associated with the high friction results is a significant increase in press time from 127 s to 327 s, which is required to achieve the same center thickness of the baseline cases. It is observed from the figure that the deviation changes significantly when friction at the interface is altered, and is more pronounced for the steep meniscus case. This

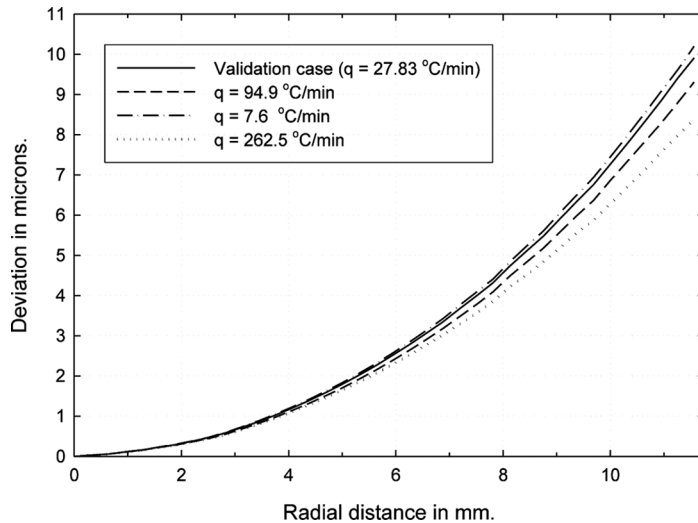
increase is due to both changes in stress and to increased heating during the longer press time.

### Effect of Thermal Properties and Cooling Rates

In this section some factors that affect temperature, other than those already studied in Figures 4–8, are considered. This is motivated by the strong thermal influences on the deviation observed in Figures 4, 6 and 8. In Figure 13 the results of a sensitivity analysis of the effect of the thermal conductivities of both glass and molds on the deviation is presented. When the thermal conductivity of glass is made unusually high (equal to that of the mold in this case), there is a small thermal gradient across the thickness of the lens and the lens will undergo nearly uniform cooling. Hence the lens shrinks as a whole without much effect from thermal stresses, which increases the deviation as shown in Figure 13(a). If the thermal conductivity of the molds is decreased such that it is close to the thermal conductivity of glass, then the molds also cool slowly. Once again a smaller thermal gradient in the lens results, the lens shrinks more uniformly without resistance from thermal stresses, which increases the deviation as shown in Figure 13(b).



**Figure 13** Sensitivity of thermal conductivity of (a) glass and (b) mold on deviation.

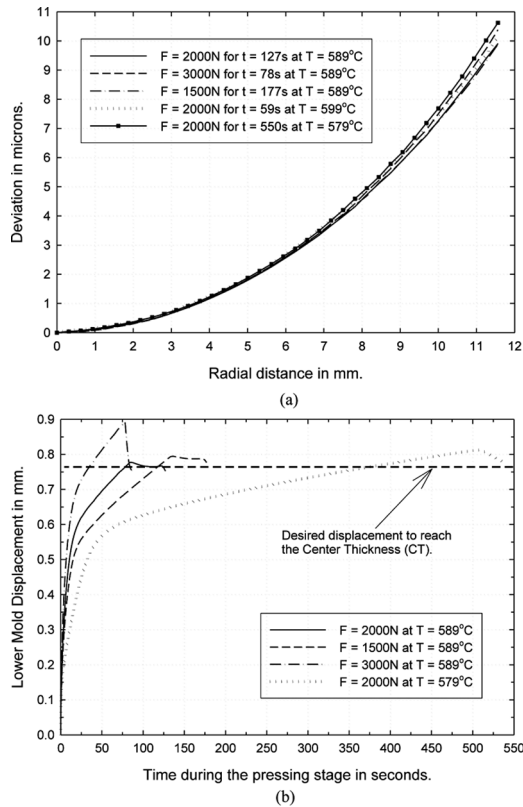


**Figure 14** Sensitivity of cooling rate during the rapid cooling stage (after the gap is created) on the final deviation for the bi-convex lens.

Because the computational model has truncated boundaries, one difficulty is the establishment of the temperature boundary conditions based on the limited temperature data obtained from the molding machine. Because of this it is important to vary these boundary conditions. In Figure 14 the sensitivity on deviation of cooling rates during the rapid cooling stage is studied since structural relaxation is cooling rate dependent. For extreme variations in the cooling rates from 7.6°C/min to 262°C/min, the change in deviation is only two microns. Hence, within a practical range of cooling rates, the deviation does not change appreciably. The same conclusion was made by Jain and Yi [2]. The trend of decrease in deviation, however small, to increasing cooling rates can be explained using the theory of structural relaxation. The volume of a glass sample which has attained equilibrium is always smaller than one in a frozen state. As the cooling rate increases, there is not enough time for the structure to reach equilibrium. Hence the structure becomes frozen with limited shrinkage which reduces deviation. On the other hand, if the cooling rate is very slow, the structure has enough time to equilibrate and hence shrinkage increases, giving more deviation.

### Effect of Molding Force and Molding Temperature

The molding force and molding temperature will now be varied to see how these important process parameters affect deviation. In this case the pressing time must also be adjusted to achieve the same center thickness, which requires a reliable characterization of stress relaxation and the associated TRS behavior. In Figure 15 for several scenarios of force and temperature, very little differences in deviation occur. However, this conclusion cannot be generalized to apply to any geometry. The lower part of the figure shows the detail of how center thickness is achieved through control of the lower mold position.



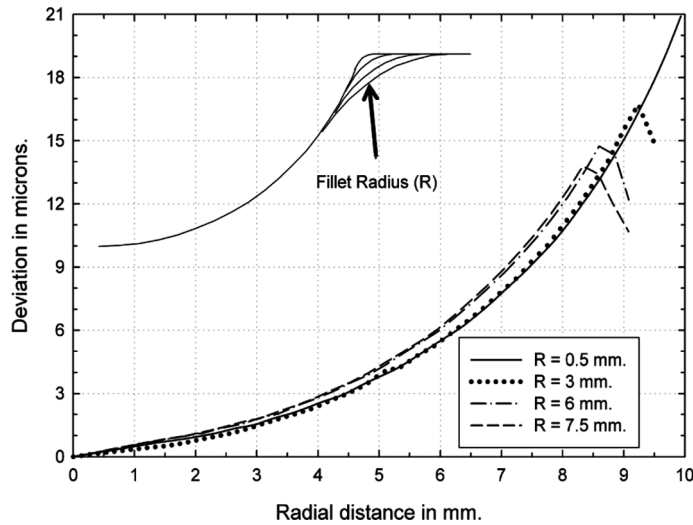
**Figure 15** Sensitivity of the combination of molding force, press time and molding temperature on the deviation for the bi-convex lens. Parameters were selected to give the same Center Thickness (CT). (a) Sensitivity of Molding force and Molding temperature on deviation and (b) Corresponding displacement of the lower mold as a function of time.

### Effect of Mold Geometry Beyond the Lens Aperture

Referring to Figure 17 in Part I of this two-part series, it was observed that during slow cooling most of the lens loses contact with the molds. When the molding force is reduced to the maintenance force, a small gap opens resulting in a pinching action from the “F1” and “F2” forces around the perimeter of both lens types. To point out the importance of this observation, the results presented in Figure 16 summarize a sensitivity analysis on the effect of the fillet radius of the lower mold beyond the lens aperture on the deviation for the steep meniscus lens. The fillet radius is defined as an insert in Figure 16. This shows how changes in the fillet radius, which result in slight changes in how the F1 and F2 forces are applied to the lens, can affect final deviation.

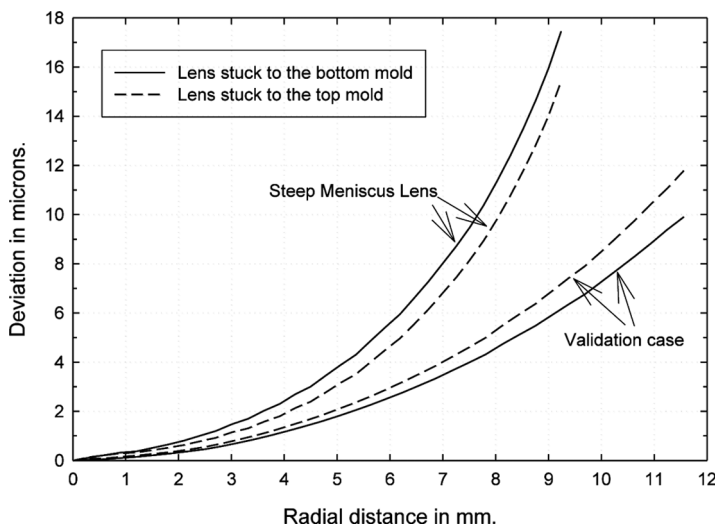
### Effect of Lens Position with Respect to the Molds after Gap Creation

One way to study the importance of gap conductance is to change the location of the lens with respect to the upper and lower molds once the gap is created. This



**Figure 16** Effect of fillet radius of the lower mold on deviation for the steep meniscus lens (see Figure 17 and related discussion in Part I). The baseline case corresponds to  $R = 0.5$  mm.

can be an issue in lens molding, as sometimes the lens sticks to the upper surface of the mold. After the slow cooling stage, a small gap of magnitude 0.15 mm is created between the lens and the molds as shown in Figure 8 of Part I. This gap is created to prevent the lens from breaking during the final cooling stage when it is brittle and no longer has the viscous mechanism to accommodate a thermal strain. In Figure 17 the deviation is plotted for both lens types when the lens “sticks” to the upper mold



**Figure 17** Change in deviation observed depending on whether the lens sticks to the top mold or lies on the bottom mold for the two different lens geometries considered.

for the entire cooling process, instead of the cases considered previously when the lens rested on the lower mold. The non-negligible difference between these two cases shows the importance of the gap conductance, and on knowing the location of the lens when the initial release from one of the molds occurs.

Also interesting is the change in trend for the two different geometries considered. When the gap location changed from above the lens to below the lens, the deviation decreased for the bi-convex lens, but increased for the steep meniscus lens. The heat transfer mechanism from glass to the molds is changed significantly depending on the location of the gap which causes the change in deviation. Furthermore, it is possible for a lens to “stick” to the upper mold surface by the creation of a vacuum, for example, and then drop before the lens is removed from the machine. In this case the deviation would be a function of the time the lens was attached to the upper surface and this time could be a very sensitive function of the upper mold shape. Therefore, the study in Figure 17 reveals a potential difficulty in mold compensation.

## DISCUSSION

A mathematical model was developed to simulate the precision lens molding process taking into account process details and the complex material behavior of glass to predict the final profile deviation of the lens with micron level accuracy. The important processing stages included in the model are heating, soaking, pressing, gap creation and cooling. The most important material behaviors of glass are the strongly temperature dependent viscoelastic behavior and structural relaxation. Structural relaxation behavior as considered in this study is temperature history dependant thermal expansion behavior and is modeled in ABAQUS with user subroutines.

The main objective of this study was to quantify the effect on deviation of the key material properties and process parameters. Those that have a large effect must be characterized accurately for modeling purposes, can be targeted in glass science studies to produce better molding glasses and should be understood by the lens designer when selecting materials or adjusting process parameters. Similarly, those that have little effect need not be characterized precisely and provide flexibility to the glass scientist and lens designer. A summary of these results is provided in Table 4.

It is important to realize that some of the glass and mold material and interface properties needed for the simulation of the molding process are not available in the literature. Although a significant experimental effort was made to have a realistic set of baseline material properties and process parameters, the material properties selected to represent L-BAL35 glass, should, from a rigorous point of view, be considered to represent a “hypothetical glass.” The principal weaknesses are believed to be the temperature dependent stress relaxation characterization and the gap conductance. In addition to experimental characterization, a significant effort was made to validate the model using two different lens geometries. These detailed studies provide an honest assessment of the validity of the model, and show that in addition to stress relaxation and gap conductance, it is possible that there are other limitations. For example, if some of the structural relaxation parameters have a slight error, the deviation can change



**Table 4** Several material properties/processing parameters that affect deviation are listed in the order of importance

Importance of sensitivity parameters studied on deviation	
Important parameters	Non-important parameters
Structural relaxation parameter, $\Delta H/R$	Thermal conductivity of glass and molds ( $\kappa$ )
Structural relaxation parameter, $\tau_0$	Structural relaxation parameter, $x$
CTE of the mold, $\alpha_{mold}$	Pressing force
TRS behavior of glass	Molding temperature
Elastic Modulus of glass, $E(T)$	Elastic modulus of the mold, $E_{mold}$
Friction, $\mu$	Structural relaxation parameter, $\beta$
Temperature at which gap is created	Specific heat, $c_p$
Cooling profile ( $q = dT/dt$ )	Gap/contact conductance, hg & decay profile
Location of gap (top or bottom)	
Fillet Radius of lower mold beyond aperture	

significantly. Based on this experience, in general it is non-trivial to have a successful validation, which requires:

1. an accurate computational model,
2. a valid characterization of the material behavior of glass, the molds and the mold/glass interface,
3. an accurate set of process parameters including temperature and molding force,
4. an accurate measurement of the final lens shape.

## CONCLUSIONS

The following important conclusions were made:

1. Deviation arises from both the thermal expansion behavior and the evolving stress state within glass during the course of molding.
2. Structural relaxation of glass is the primary reason for the deviation in the molded lens. Specifically, the activation energy constant  $\Delta H/R$  and time constant parameter  $\tau_0$  are the key parameters of structural relaxation that affect deviation. Hence, glasses with different values of these parameters must be compensated differently. Clearly from a computational point of view, it is essential to have the structural relaxation parameters well defined to predict the deviation within tolerance. The thermal expansion coefficient of the mold material is also important.
3. Internal stresses also affect the deviation. The evolving internal stresses within glass can be affected by changing
  - a. the TRS behavior of glass at temperatures close to the molding temperature
  - b. friction at the glass/mold interface,
  - c. the time/temperature at which the gap is created,

The most critical stage in the process is the opening of the gap when the maintenance force is removed, as this results in a sudden change in deviation

for some lens geometries and process conditions. For these cases it is critical to determine accurately the stress state in the glass, which requires an accurate characterization of stress relaxation, including both the master curve, the TRS material behavior near the molding temperature and the elastic modulus. Other important factors include the molding temperature, the force profile and the cooling profiles. These are readily available from the machine after the molding process is completed and in this study were assumed to be correct.

4. The deviation is sensitive to the positioning of the lens within the gap created between the two cooling stages. After the creation of this gap, the lens usually rests on the bottom mold, but sometimes “sticks” to the upper mold. In some cases a lens will still be attached to the upper mold at room temperature, but it is conceivable that the lens drops at some point during the cooling cycle. Due to differences in heat transfer, as seen in Figure 17, the simulations predict a non-negligible difference in deviation between the two extreme situations, which might be one of the reasons for difficulty in mold compensation. A slightly different mold shape could cause the lens to drop at different times.
5. The temperature of the glass preform is typically lower than the readings in the temperature sensors of the precision lens molding machine. Furthermore, this temperature lag is more pronounced when a vacuum is present. Therefore, it is extremely important to be able to predict temperature accurately and to have stress relaxation accurately characterized with respect to temperature.
6. This investigation does not take into account, for example, the effect of a stress state that could lead to lens failure. While this sensitivity analysis might show that final size and shape are the same for two cases, one case could actually crack the lens while the other could result in an acceptable lens.

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